Landau-Zener Transitions in a Noisy Environment

Переходы Ландау-Зинера в шумящей среде

Валерий Покровский (Valery Pokrovsky)
Texas A&M University, College Station, Texas, USA
and
Институт Теоретической Физики им. Ландау РАН
Черноголовка Московской обл.
Российская Федерация

Outline

- Introduction to Landau-Zener transitions
- Some applications of the Landau-Zener theory
- Why and when noise is substantial
- Description of noise
- Transitions in the presence of fast noise
- Interaction of the transverse and longitudinal noise
- Conclusions

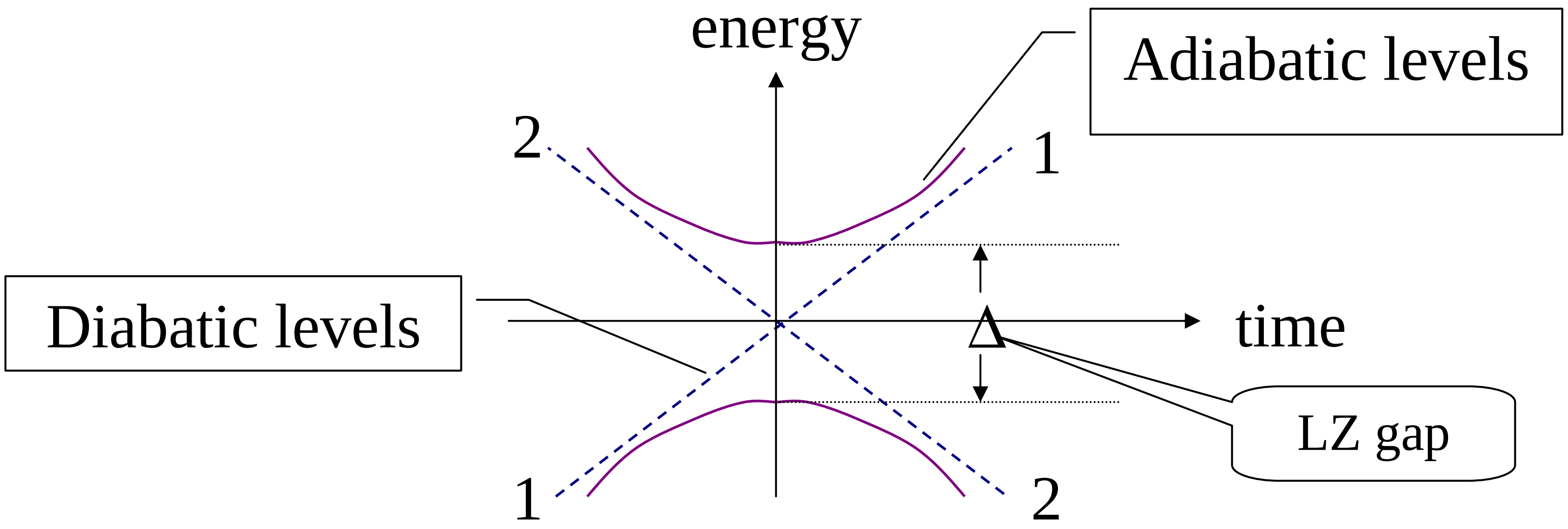
L.D. Landau, Phys. Z. Sowietunion, **2**, 46 (1932)

C. Zener, Proc. R. Soc. A**137**, 696 (1932)

E.C.G. Stückelberg, Helv. Phys. Acta **5**, 369 (1932)

E. Majorana, Nuovo Cimento **9**, 43 (1932)

Introduction: LZ theory



Avoided level crossing (Wigner-Neumann theorem)

Schrödinger equations

$$i\dot{a}_1 = E_1(t)a_1 + \Delta a_2$$
$$i\dot{a}_2 = \Delta^* a_1 + E_2(t)a_2$$

$$E_2(t) - E_1(t) = \Omega(t); \quad \hbar = 1$$
$$\Omega(t) = \dot{\Omega} t$$

Diabatic levels:

$$E_2 = -E_1 = \dot{\Omega} t / 2$$

Adiabatic levels:

$$E_\pm = \pm \sqrt{\left(\frac{E_1 - E_2}{2} \right)^2 + |\Delta|^2}$$

LZ parameter:

$$\gamma = \frac{\Delta}{\hbar \sqrt{\dot{\Omega}}}$$

$\gamma \square 1$

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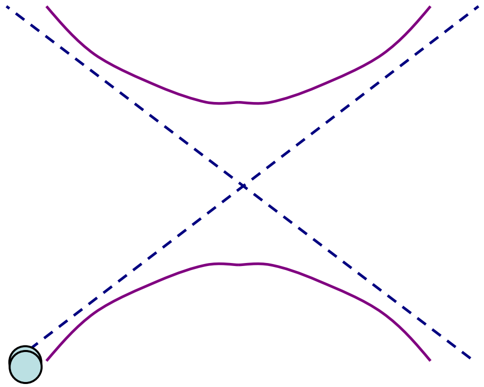


LZ transition matrix

$$U=\begin{pmatrix}\alpha & \beta\\ -\beta^{*} & \alpha^{*}\end{pmatrix}\qquad |\alpha|^{2}+|\beta|^{2}=1$$

Amplitude to stay at the same diabatic level (surviving amplitude)

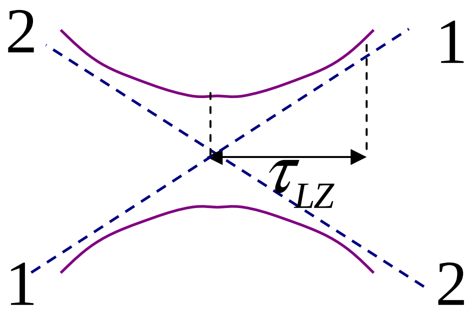
$$\alpha=e^{-\pi\gamma^{2}}$$

Amplitude of transition

$$\beta=-\frac{\sqrt{2\pi}\,\exp\left(-\frac{\pi\gamma^{2}}{2}+i\frac{\pi}{4}\right)}{\gamma\Gamma\left(-i\gamma^{2}\right)}$$

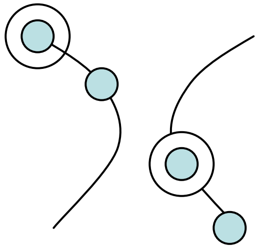
LZ transition time:

$$\tau_{LZ}=\frac{\Delta}{\dot{\Omega}}$$



Some applications of the Landau-Zener theory

Electronic transitions at atomic collisions



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Proc. Natl. Acad. Sci. USA
Vol. 77, No. 6, pp. 3105–3109, June 1980
Chemistry

Role of the chlorophyll dimer in bacterial photosynthesis

(electron transfer/charge separation/light energy storage)

ARIEH WARSHEL

Department of Chemistry, University of Southern California, Los Angeles, California 90007

Communicated by Martin D. Kamen, March 18, 1980

ABSTRACT The role of a special dimer (D) of bacteriochlorophyll molecules in bacterial photosynthesis was examined by calculations of the rates of electron transfer reactions in a system of the dimer and a bacteriopheophytin (BPh) molecule. It was found that the dependence of the potential surfaces of D on the distance between the monomers allows a fast light-induced electron transfer from D to BPh but only a slow back reaction (reduction of D^+ by BPh^-). The same potential surfaces allow efficient reduction of D^+ by cytochrome *c*. Possible advantages of greatly different values of the electronic matrix elements for the forward and back reactions are pointed out. It is suggested that the electrostatic interaction between D^+ and an ionized group of the protein might play an important role in the photosynthetic reaction.

classical trajectory approach of ref. 8. For small σ_{AB}, when the diabatic approximation is valid this gives essentially the Landau–Zener transition probability (8) which, when substituted into the rate expression in transition rate theory, gives exactly the same expression as Eq. 2.

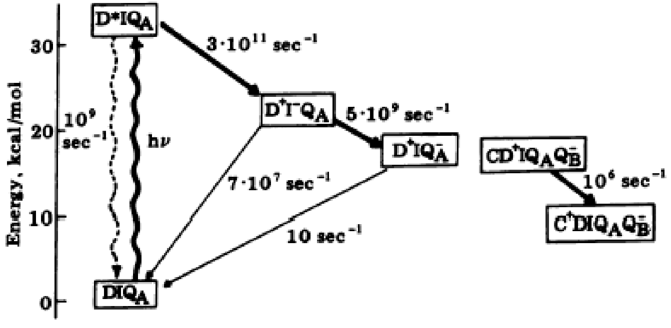
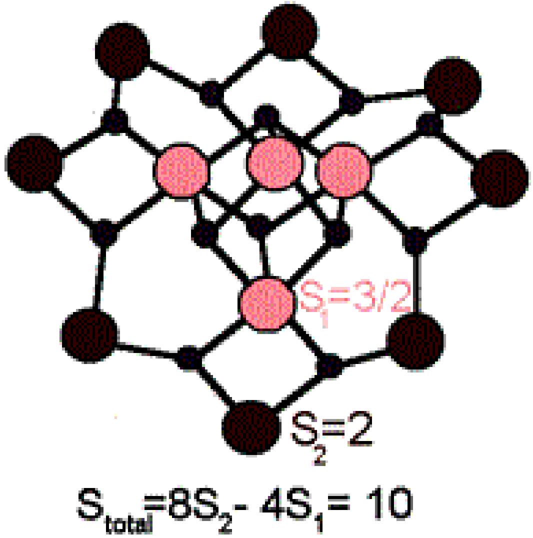


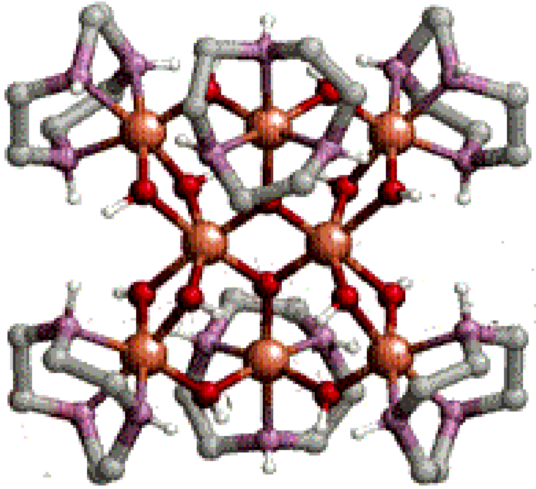
FIG. 1. Rates of electron transfer along different possible reaction

Molecular magnets

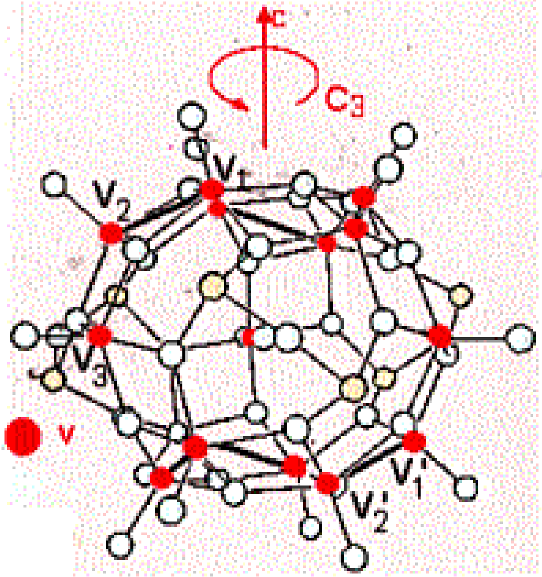
- $S = 10$: Mn_{12}, Fe_8. $S = 1/2$: V_{15}.



Mn_{12}

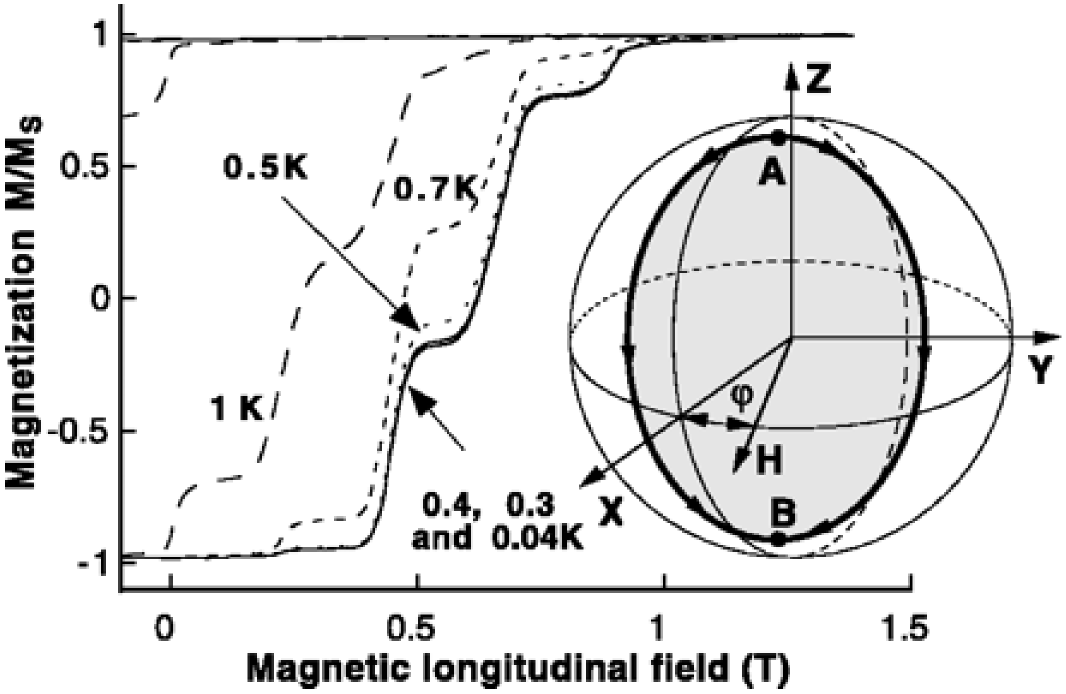


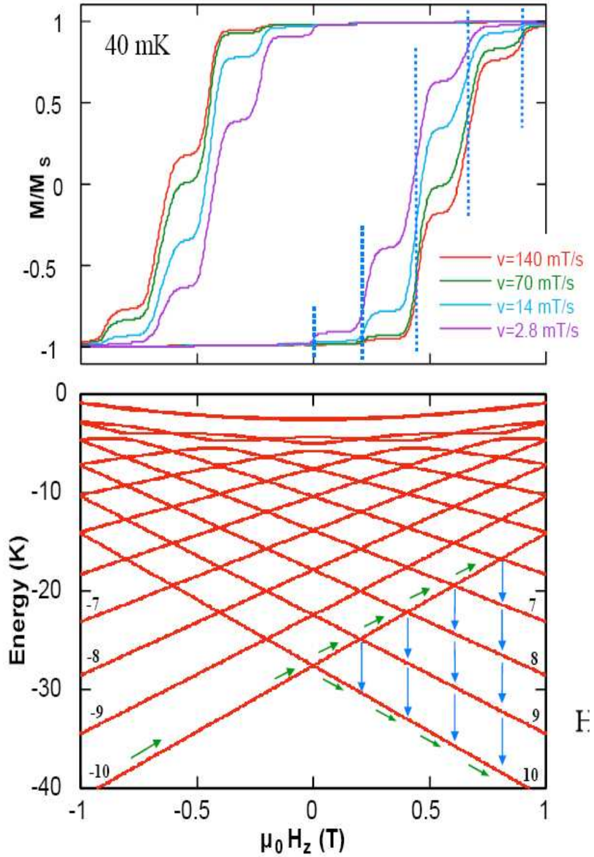
Fe_8



V_{15}

Spin reversal in nanomagnets



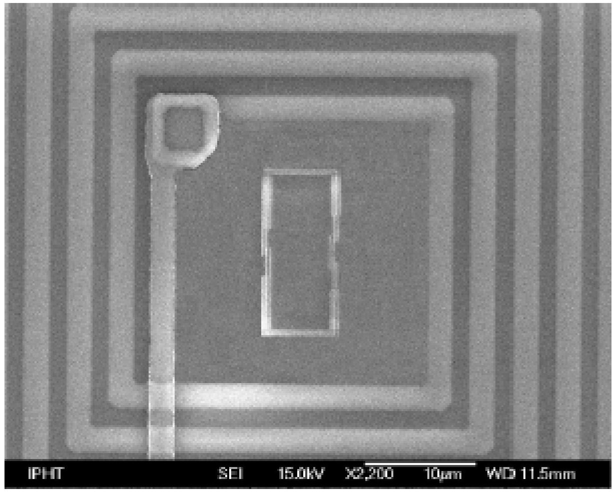


W. Wernsdorfer and R. Sessoli, Science **284**, 133 (1999)

Observation of macroscopic Landau–Zener transitions in a superconducting device

A. Izmalkov[1,2], M. Grajcar[1,3], E. Il'ichev[1](*), N. Oukhanski[1], Th. Wagner[1], H.-G. Meyer[1], W. Krech[4], M. H. S. Amin[5], Alec Maassen van den Brink[5] and A. M. Zagoskin[5,6]

[1] *Institute for Physical High Technology, P.O. Box 100239, D-07702 Jena, Germany*
[2] *Moscow Engineering Physics Institute (State University), Kashirskoe shosse 31, 115409 Moscow, Russia*
[3] *Department of Solid State Physics, Comenius University, SK-84248 Bratislava, Slovakia*
[4] *Friedrich Schiller University, Institute of Solid State Physics, D-07743 Jena, Germany*
[5] *D-Wave Systems Inc., 320-1985 West Broadway, Vancouver, B.C., V6J 4Y3 Canada*
[6] *Physics and Astronomy Dept., The University of British Columbia, 6224 Agricultural Rd., Vancouver, B.C., V6T 1Z1 Canada*



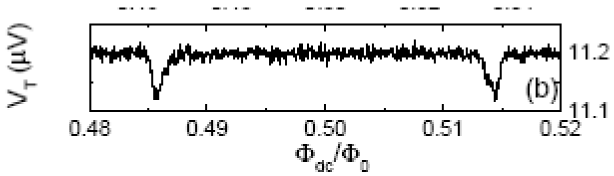
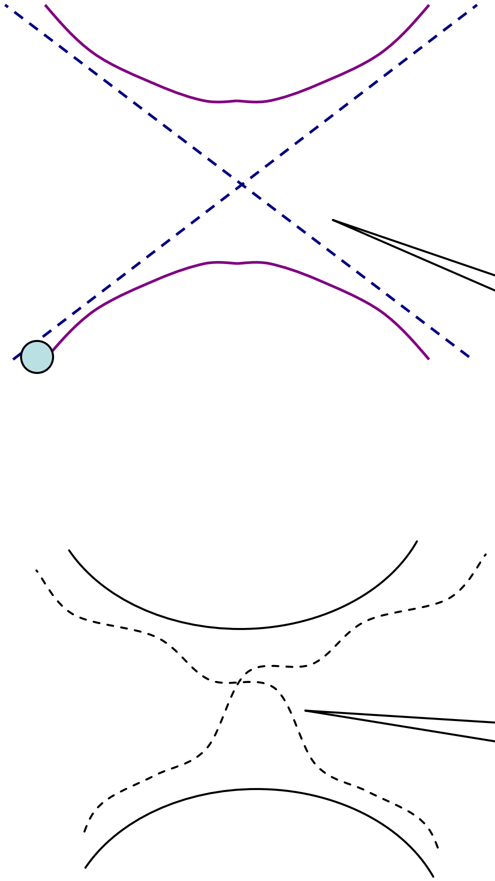


Fig. 2 – Electron micrograph of the qubit at the centre of the tank coil.

Why and when noise is substantial?

Controllable switch between states
for quantum computing (qubit):

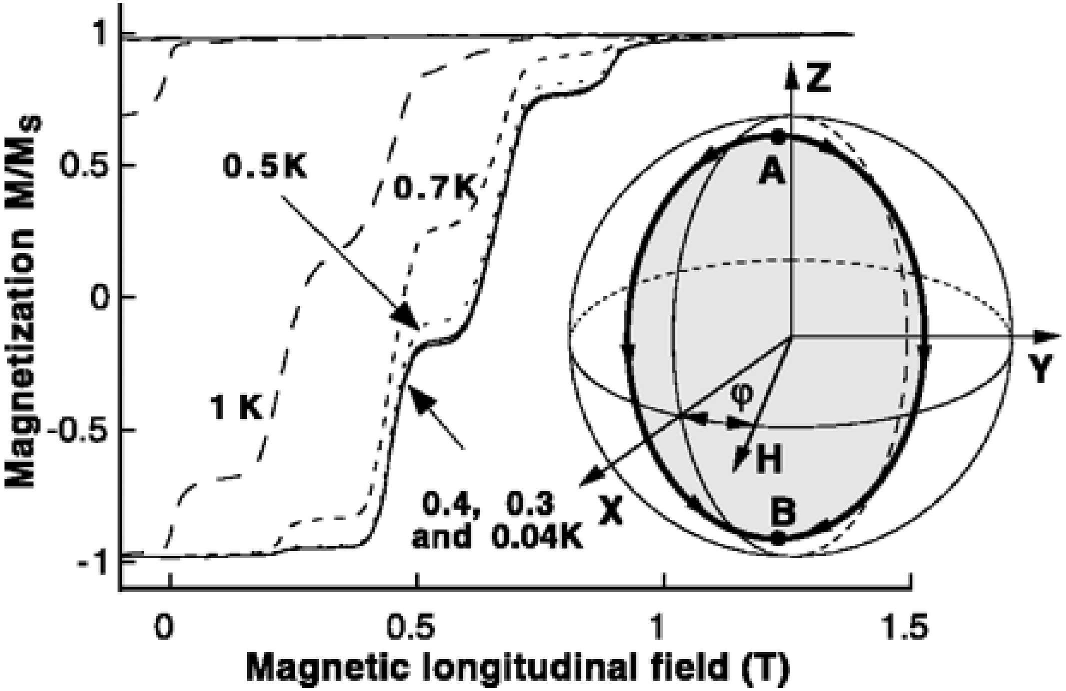


The noise introduces mistakes to the switch work.

Transverse noise

Longitudinal noise
create decoherence

Molecular magnets



Thernal noise changes the hysteresis loop at T>0.5 K

History

Longitudinal noise

Y. Kayanuma, J. Phys. Soc. Jpn. **54**, 2087 (1985)

Y. Gefen, E. Ben-Jacob, and A.O. Caldeira, Phys. Rev B **36**, 2770 (1987)

P. Ao and J. Rammer, Phys. Rev. B **43**, 5397 (1991)

Y. Kayanuma and H. Nakamura, Phys. Rev. B **57,** 13099 (1998)

Classical transverse noise

Y. Kayanuma, J. Phys. Soc. Jpn. **53**, 108 (1984)

V.L. Pokrovsky and N.A. Sinitsyn, Phys. Rev. B **67**, 144303 (2003).

Quantum fast noise

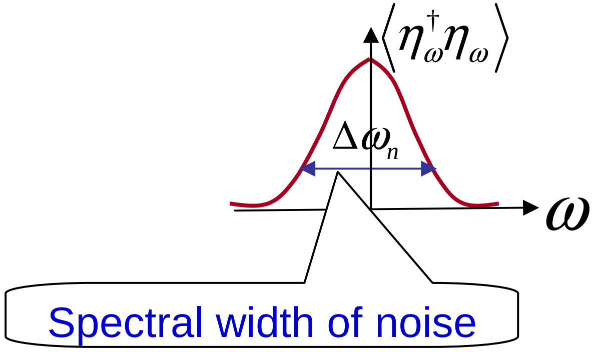
V.L. Pokrovsky and D. Sun, Phys. Rev. B **76**, 024310 (2007).

Description of noise

- Transverse or longitudinal

$$\eta(t) = \eta_l(t)\sigma_z + \eta_t(t)\sigma_x$$

- Fast or slow



Fast noise:

$$(\Delta\omega_n)^{-1} = \tau_n \ll \tau_{LZ}$$

- Strong or weak

Strong noise: $\langle \eta_{l,t}^2(t) \rangle \gg \hbar^2 \dot{\Omega}$

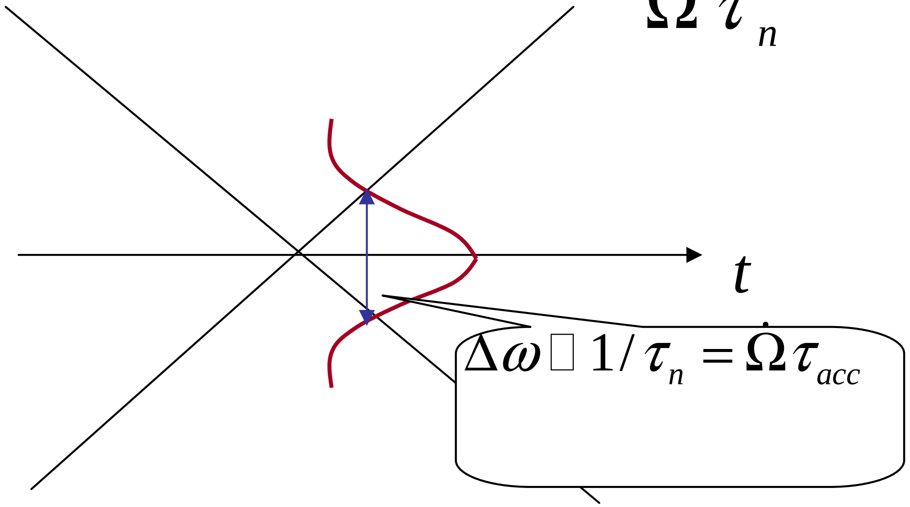
- Quantum or classical

Quantum noise: $\langle \eta_\alpha(t)\eta_\beta(t') \rangle \neq \langle \eta_\beta(t')\eta_\alpha(t) \rangle$

- Thermal or non-thermal

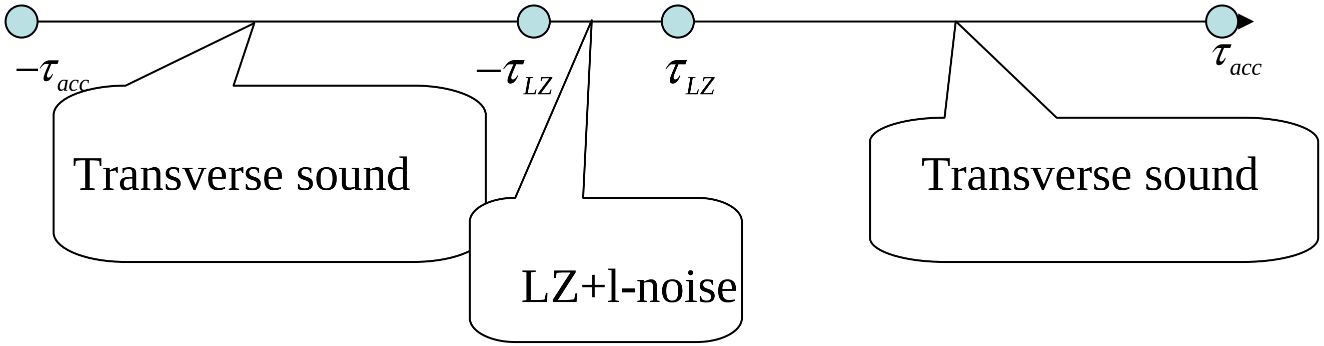
Thermal noise: $\langle \eta_\omega^\dagger \eta_\omega \rangle = e^{-\hbar\omega/T} \langle \eta_\omega \eta_\omega^\dagger \rangle$

Accumulation time: $\tau_{acc} = \dfrac{1}{\dot{\Omega}\,\tau_n} \square \;\; \tau_n$



$\Delta\omega \square 1/\tau_n = \dot{\Omega}\tau_{acc}$

Time separation of LZ and transverse-noise-induced transitions



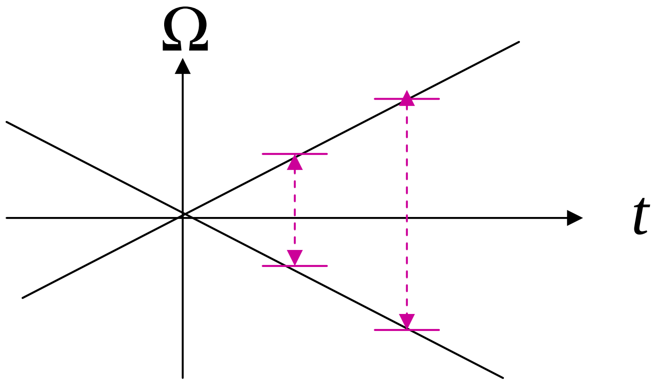
Transverse sound

LZ+l-noise

Transverse sound

Fast transverse noise in 2-level systems:

Transition is produced by that spectral component of noise, whose frequency is in resonance with instantaneous frequency of the LZ 2-level system.



Transition probability measures the spectrum of noise

Master equation

$$\dot{n}_1 = -\left\langle \eta^{\dagger}_{-\Omega(t)} \eta_{-\Omega(t)} \right\rangle n_1 + \left\langle \eta_{\Omega(t)} \eta^{\dagger}_{\Omega(t)} \right\rangle n_2 \qquad n_1 + n_2 = 1$$

Master equation:

$$n_1 + n_2 = 1 \qquad s_z = \frac{n_1 - n_2}{2} \qquad n_{1,2} = \frac{1}{2} \pm s_z$$

$$\frac{ds_z}{dt} = \left[-s_z \left(\left\langle \eta \eta^\dagger \right\rangle_{|\Omega|} + \left\langle \eta^\dagger \eta \right\rangle_{-|\Omega|} \right) + \operatorname{sign}(\Omega) \left(\left\langle \eta \eta^\dagger \right\rangle_{|\Omega|} - \left\langle \eta^\dagger \eta \right\rangle_{-|\Omega|} \right) \right]_{\Omega = \Omega(t)}$$

Classical limit: $\left\langle \eta \eta^\dagger \right\rangle_{\Omega} = \left\langle \eta^\dagger \eta \right\rangle_{-\Omega}$

Adiabatic limit $s_z(t) = -\operatorname{sign}(\Omega) \dfrac{\left\langle \eta \eta^\dagger \right\rangle_{|\Omega|} - \left\langle \eta^\dagger \eta \right\rangle_{-|\Omega|}}{\left\langle \eta \eta^\dagger \right\rangle_{|\Omega|} + \left\langle \eta^\dagger \eta \right\rangle_{-|\Omega|}}$

Equilibrium: $s_z(t) = -\tanh \dfrac{\Omega(t)}{2T}$

New dimensional parameter: $\gamma_t^2 = \dfrac{\langle \eta_t^2 \rangle}{\dot{\Omega}}$

$\gamma_t \; \square \; 1 \Longrightarrow$ adiabatic regime for noise transitions

$$s_z(t) = -\mathrm{sign}(\Omega)\frac{\langle \eta\eta^\dagger \rangle_{|\Omega|} - \langle \eta^\dagger\eta \rangle_{-|\Omega|}}{\langle \eta\eta^\dagger \rangle_{|\Omega|} + \langle \eta^\dagger\eta \rangle_{-|\Omega|}}$$

$\gamma_t \; \square \; 1 \Longrightarrow$ noise transitions are negligible – pure LZ transition

New time scale: decoherence time: $\tau_{dec} = \left(\langle \eta_\alpha{}^2 \rangle \tau_n \right)^{-1}$

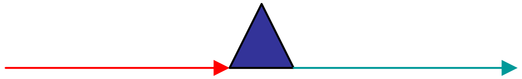
Survival probability:

$$P_{1\to1} = \frac{1}{2}\left[1 + e^{-2\pi\gamma_t^2}\left(2e^{-2\pi\gamma^2} - 1\right)\right] + \frac{\pi}{\dot{\Omega}}\int_0^\infty d\Omega\, G(\Omega)\, e^{-\frac{2\pi}{\dot{\Omega}}\int_\Omega^\infty F(\omega)d\omega}\left[\left(2e^{-2\pi\gamma^2} - 1\right)e^{-\frac{4\pi}{\dot{\Omega}}\int_0^\Omega F(\omega)d\omega} - 1\right]$$

$$F(\Omega) = \langle \eta\eta^\dagger \rangle_\Omega + \langle \eta^\dagger\eta \rangle_{-\Omega};\; G(\Omega) = \langle \eta\eta^\dagger \rangle_\Omega - \langle \eta^\dagger\eta \rangle_{-\Omega}$$

Renormalization of the LZ gap

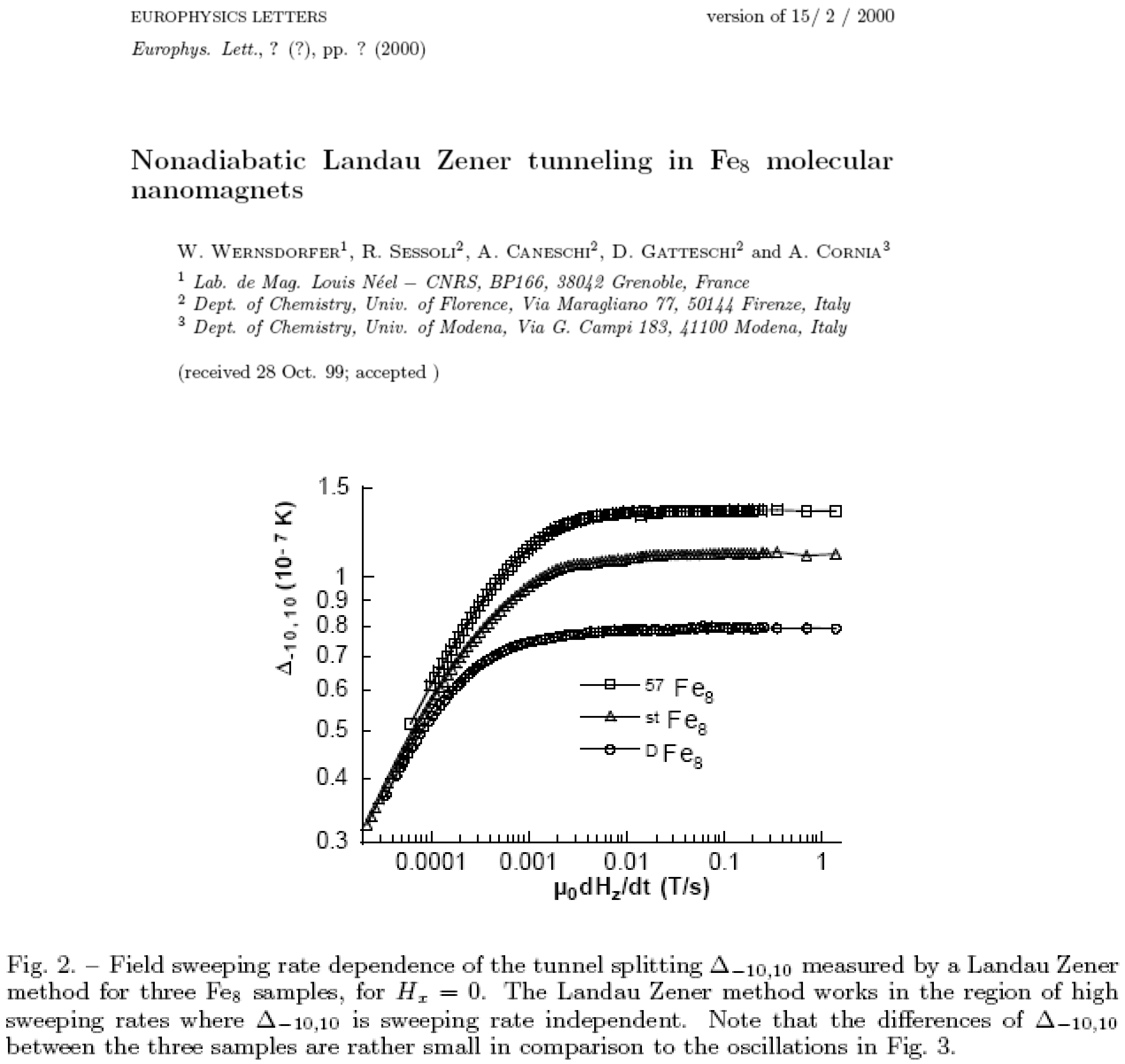
Correlated transverse and longitudinal sound produces almost instantaneous transition between the states of the 2-state system exactly as LZ gap Δ does.



$$\Delta \to \tilde{\Delta} = \Delta + i\int_0^{\infty} \left\langle \left[u_{\perp}(t), u_{\parallel}(0) \right] \right\rangle dt$$

Renormalized gap does not depend on temperature.

Renormalzation is isotopically sensitive (Wernsdorfer et al.)



EUROPHYSICS LETTERS version of 15/ 2 / 2000

Europhys. Lett., ? (?), pp. ? (2000)

Nonadiabatic Landau Zener tunneling in $\mathbf{Fe_8}$ molecular nanomagnets

W. Wernsdorfer[1], R. Sessoli[2], A. Caneschi[2], D. Gatteschi[2] and A. Cornia[3]

[1] *Lab. de Mag. Louis Néel – CNRS, BP166, 38042 Grenoble, France*
[2] *Dept. of Chemistry, Univ. of Florence, Via Maragliano 77, 50144 Firenze, Italy*
[3] *Dept. of Chemistry, Univ. of Modena, Via G. Campi 183, 41100 Modena, Italy*

(received 28 Oct. 99; accepted)

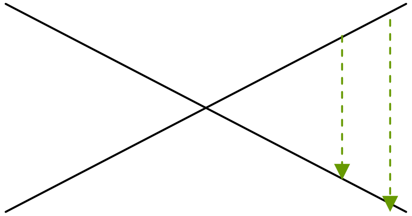
Fig. 2. – Field sweeping rate dependence of the tunnel splitting $\Delta_{-10,10}$ measured by a Landau Zener method for three Fe$_8$ samples, for $H_x = 0$. The Landau Zener method works in the region of high sweeping rates where $\Delta_{-10,10}$ is sweeping rate independent. Note that the differences of $\Delta_{-10,10}$ between the three samples are rather small in comparison to the oscillations in Fig. 3.

Zero temperature

Survival probability

$$P_{1\to 1} = \exp\left[-2\pi\left(\tilde{\gamma}^2 + \gamma_t^2\right)\right]$$

$$\tilde{\gamma}^2 = \frac{\tilde{\Delta}^2}{\dot{\Omega}}$$



Only spontaneous emission is allowed

Exact calculation: no assumptions on strength of noise and short correlation time

M. Wubs, K. Saito, S. Köhler, P. Hänggi, and Y.Kayanuma, Phys. Rev. Lett. **97**, 200404 (2006).

Conclusions

- Transitions induced by transverse noise are accumulated during a long time $\tau_{acc} = \left(\dot{\Omega}\tau_n\right)^{-1}$
- The LZ gap induces transitions during a shorter time $\tau_{LZ} = \Delta / \dot{\Omega}$
- The longitudinal noise is effective during the same time
- The coherence is destroyed during the longest time $\tau_{dec} = \left(\left\langle u^2 \right\rangle \tau_n\right)^{-1}$
- Within the accumulation time the transition probability obey the Master equations if noise is moderately strong
- The correlation of longitudinal and transverse noise leads to renormalization of the LZ gap, which explains its isotopic variation and violation of selection rule in molecular magnets